Monte Carlo renormalization group of dilute 3D Ising dynamics

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A computer simulations by the Monte-Carlo method of the critical dynamics of the diluted 3D Ising model are considered for samples with spin concentrations p_s=0.95, 0.8, 0.6 and 0.5. There is reason to believe that the influence of quenched impurities on the critical dynamics will be seen more clearly than in the equilibrium state, because of the specific conservation laws.

The disordered Ising model is specified by a system of spins $S_i = \pm 1$ with concentration p, which are associated with $N=pL^d$ sites of a d-dimensional lattice (L is length scale of the lattice). This gives $p2^N$ possible configurations $\{S\}$, with energy $E=-J\sum p_i\, p_j\, S_i S_j$, where the sum runs over nearest neighbours only, J represents spin interaction energy, and p_i are quenched site disorder variables. We consider a ferromagnetic system with $J<0$.

The use of the Metropolis kinetics makes it possible to immediately realize the dynamics of the Ising model with a relaxation of magnetization $m(t)= \sum_i^N S_i/N$ to the equilibrium value determined by the thermostat temperature T. In the simulation of the critical dynamics the initial state of the system is chosen with all spins parallel ($m(0)=1$) and with a temperature equal to the critical temperature. The critical temperature T_c for dilute magnetic materials is a function of the sin concentration p_s. It decreases with increasing p and vanishes at the threshold concentration p_c. For cubic lattice of Ising spins we would have $p_c \approx 0.3117$ and $T_c(0.95) \approx 4.26267$, $T_c(0.8) \approx 3.49948$, $T_c(0.6) \approx 2.42413$, $T_c(0.5) \approx 1.84509$ in units of J/k [1]. To determine the dynamical exponent z, which characterizes the critical slowing of the relaxation time of the system, $t_c \sim |T-T_c|^{-z\nu}$, we have used here the Monte-Carlo method, combined with the dynamical renormalization-group method. For this, the system was partitioned into blocks, where a block b^d of neighboring spins was replaced by a single spin whose direction is determined by the direction of most spins in the block. The redefined spin system forms a new lattice with magnetization m_b. We assume that the magnetization of the initial lattice relax to some value m_1, over a time t_b. Then by using two systems with block size b and b' and determining the relaxation times t_b and $t_{b'}$ of the block magnetization m_b and $m_{b'}$ reach to the same value m_1., the dynamic exponent z can be determined from the relation $t_b/t_{b'} = (b/b')^z$ or $z=\ln(t_b/t_{b'})/\ln(b/b')$ in the limit of sufficient large b and $b' \to \infty$.

We applied this algorithm to impure systems with dimensions of 144^3 and impurity concentrations presented above. The size of the system made it possible to partition into blocks with sizes *b=2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36.* The procedure of blocks partitioning of the initial spin and impurity configurations was implemented on the basis of the criterion of spin connectivity [2]. A relaxation simulating procedure consisting of 10000 Monte Carlo steps per spin was perfomed for each with 1000 runs with different impurity configurations over which the function $m_b(t)$ was averaged.

We obtained the sets of the exponent z_b values corresponding to the different b. The revealed dependence of z on b made extrapolation to the case $b \to \infty$ possible, assuming that $z_b = z_{b=\infty}$ + const $b^{-\Omega}$. The resulted values of the dynamic exponent $z = z_{b=\infty}$ are z(0.95)=2.18 ± 0.03, z(0.8)=2.24 ± 0.03, z(0.6)=2.70 ± 0.05 and z(0.5)=2.78 ± 0.08. These values of the dynamic exponent confirm the steps-like universality law, for diluted 3D magnetic materials which was suggested at first in paper [2]. According to it, the critical behaviour of slightly and strong disordered systems divided by threshold of impurity percolation belongs to different classes of critical universality with different sets of exponents.

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